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Dynamics of solitons in multicomponent long wave–short wave resonance interaction system

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Abstract. In this paper, we study the formation of solitons, their propagation and collision behaviour in an integrable multicomponent (2+1)-dimensional long wave–short wave resonance interaction (M-LSRI) system. First, we briefly revisit the earlier results on the dynamics of bright solitons and demonstrate the fascinating energy exchange collision of bright solitons appearing in the short-wave components of the M-LSRI system. Then, we explicitly construct the exact one- and two-multicomponent dark soliton solutions of the M-LSRI system by using the Hirota's direct method and explore its propagation dynamics. Also, we study the features of dark soliton collisions.

Keywords. Long wave–short wave resonance interaction; Hirota's bilinearization method; bright and dark solitons; soliton collision.

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1. Introduction

Nonlinear waves appearing in multicomponent nonlinear evolution equations governing the dynamics of various interesting physical systems display intriguing propagation and collision properties. The nonlinear waves, mainly solitons, which arise as the solutions of integrable nonlinear equations show interesting collision features due to their remarkable stability property and find innumerable applications in different areas of science and technology [1]. Particularly, higher-dimensional multicomponent systems admit various localized structures like solitons, vortex solitons, dromions, and so on. These multicomponent higher-dimensional solitons (HDSs) have attracted our interest to pursue a systematic study on their propagation and intriguing collision dynamics which will be of physical significance in different contexts of nonlinear science. In order to unearth the features of HDS, in this study we consider the following set of integrable nonlinear evolution equations describing the resonance interaction of multiple short waves (SWs) of high frequency with a long wave (LW) of low frequency, which is referred to as (2 + 1)-dimensional multicomponent long wave–short wave resonance interaction (M-LSRI) system,

$$i(S_t^{(\ell)} + S_y^{(\ell)}) - S_{xx}^{(\ell)} + LS^{(\ell)} = 0, \qquad \ell = 1, 2, 3, ..., M, \tag{1a}$$

$$L_t = 2\sum_{\ell=1}^{M} |S^{(\ell)}|_x^2, \tag{1b}$$

where $S^{(\ell)}$ represents the ℓth SW, L indicates the LW and the subscripts represent the partial derivatives with respect to the evolutional coordinate t and the spatial coordinates (x and y). In the above (2+1)D M-LSRI system, '2' stands for the two spatial dimensions (x and y), '1' stands for the evolutional coordinate t and M represents the number of SW components of the system.

The resonance interaction of long wave and short waves takes place when there occurs an exact (approximate) balance between the phase velocity of a LW ($v_{\rm p}$) and the group velocity of multiple SWs ($v_{\rm g}$), i.e., $v_{\rm p} \simeq v_{\rm g}$ [2–5]. Such LSRI phenomenon in different types of one- and two-dimensional nonlinear systems has been analysed extensively in the literature (for a detailed information, see [6–15] and references therein). The above-mentioned (2+1)D M-LSRI system (1) is one such model which supports several interesting dynamical features. In the context of nonlinear optics, system (1) can be derived from a set of two-dimensional multiple coupled nonlinear Schrödinger-type equations, when long wave–short wave resonance takes place [6,7].

To highlight the historical perspectives of the considered system, we wish to point out that the simplest form of (1) i.e., the one-component ($M = 1$) two-dimensional LSRI system has been obtained by using a perturbation method in a two-layer fluid model and soliton solutions were constructed by applying the Hirota method [8]. Later, in [9], special bright multisoliton solutions in Wronskian form were obtained for the two-component ($M = 2$) LSRI system and the Painlevé integrability analysis of that two-component LSRI equation was carried out in [10] with special dromion solutions. The more general bright multisoliton solution of the (2+1)D M-LSRI system (1) was obtained in [11] and fascinating energy sharing (shape changing) collision of bright solitons has been explored. Also, the propagation and collision dynamics of bright multisoliton bound states and mixed (bright–dark) solitons of system (1) have been discussed in [12] and [13], respectively. Recently, new integrable generalizations of the M-LSRI system (1) in (1+1)D, referred to as M-Yajima–Oikawa system, and in (2+1)D have been reported in [14] and [15], respectively.

The objective of this paper is to showcase the dynamics of bright and dark solitons of the M-LSRI system. We obtain the bilinear equations of the M-LSRI system (1) by using the Hirota's direct method in §2. In §3, we revisit the earlier studies on the dynamics of bright multisoliton of system (1). Then, we construct the one- and two-dark soliton solutions of the M-LSRI system (1) and explore its collision dynamics in §4. We summarize the main results in §5.

2. Bilinear equations of the M-LSRI system (1)

Hirota's bilinearization method [16] is one of the most efficient analytical tools to construct soliton solutions of integrable nonlinear evolution equations due to its algebraic nature. In this section, to obtain the soliton solutions of the M-LSRI system (1) by applying the Hirota's method, we transform the nonlinear equations (1) into a set of bilinear equations using the following transformation:

$$S^{(\ell)} = \frac{g^{(\ell)}}{f}, \quad \ell = 1, 2, ..., M, \tag{2a}$$

$$L = -2\frac{\partial^2}{\partial x^2}(\ln f), \tag{2b}$$

where $g^{(\ell)}$ and f are arbitrary complex and real functions of x, y and t, respectively. Then we can write eqs (1) as a set of bilinear equations:

$$\left(i(D_t + D_y) - D_x^2\right) g^{(\ell)} \cdot f = 0, \qquad \ell = 1, 2, 3, ..., M, \tag{3a}$$

$$(D_x D_t - 2\lambda) f \cdot f = -2\sum_{\ell=1}^{M} \left|g^{(\ell)}\right|^2. \tag{3b}$$

In eqs (3), λ is an unknown constant to be determined, D_x, D_y and D_t are the standard Hirota's D-operators [16]. For $\lambda = 0$, eqs (3) admit bright soliton solutions with zero background, while for the general case ($\lambda \neq 0$) eqs (3) can exhibit bright–dark and dark–dark soliton solutions. In this paper, we briefly revisit some interesting results of our earlier study on the propagation and collision dynamics of bright multisolitons [11]. Then we construct the dark soliton solutions of the M-LSRI system (1) and investigate their dynamics in detail.

3. Bright multisoliton solution and collision dynamics: An overview

We have obtained the explicit form of more general bright n-soliton solution, for arbitrary n, by applying the Hirota's method (see [11]). For this purpose, the power series expansion of variables $g^{(\ell)}$ and f are expressed as

$$g^{(\ell)} = \sum_{j=1}^{n} \chi^{2j-1} g_{2j-1}^{(\ell)}, \quad \ell = 1, 2, ..., M$$

and

$$f = 1 + \sum_{j=1}^{n} \chi^{2j} f_{2j},$$

respectively. On substituting $g^{(\ell)}$ and f in the bilinear eqs (3) and solving the resulting equations arising at different powers of χ, we get the exact expression for $g^{(\ell)}$ and f in the form of Gram determinants as

$$g^{(\ell)} = \begin{vmatrix} A & I & \phi \\ -I & B & \mathbf{0}^T \\ \mathbf{0} & a_\ell & 0 \end{vmatrix}, \quad f = \begin{vmatrix} A & I \\ -I & B \end{vmatrix}. \tag{4a}$$

Then from eqs (2) and (4a), the bright n-soliton solution can be written as

$$S^{(\ell)} = \frac{g^{(\ell)}}{f}, \quad \ell = 1, 2, ..., M \tag{4b}$$

and

$$L = -2\frac{\partial^2}{\partial x^2}(\ln f). \tag{4c}$$

In eq. (4a), I and $\mathbf{0}$ represent identity matrix and null matrix of dimensions $(n \times n)$ and $(1 \times n)$, respectively, A and B are square matrices of dimension $(n \times n)$ with elements

$$A_{ij} = \frac{e^{\eta_i + \eta_j^*}}{k_i + k_j^*},$$
$$B_{ij} = \kappa_{ji} = \frac{-\psi_i^\dagger \psi_j}{(\omega_i^* + \omega_j)} \equiv \frac{-\sum_{\ell=1}^{M} \alpha_j^{(\ell)} \alpha_i^{(\ell)*}}{(\omega_i^* + \omega_j)}, \quad i, j = 1, 2, \ldots, n, \tag{4d}$$

a_ℓ, ψ_j and ϕ are block-matrices of dimensions $(1 \times M)$, $(M \times 1)$ and $(n \times 1)$, respectively, with elements

$$a_\ell = -\left(\alpha_1^{(\ell)}, \alpha_2^{(\ell)}, \ldots, \alpha_n^{(\ell)}\right), \quad \psi_j = \left(\alpha_j^{(1)}, \alpha_j^{(2)}, \ldots, \alpha_j^{(M)}\right)^T$$

and

$$\phi = \left(e^{\eta_1}, e^{\eta_2}, \ldots, e^{\eta_n}\right)^T,$$

where

$$\eta_j = k_j x - (ik_j^2 + \omega_j) y + \omega_j t, \quad j = 1, 2, \ldots, n, \quad \ell = 1, 2, 3, ..., M.$$

Here k_j, ω_j and $\alpha_j^{(\ell)}$, $j = 1, 2, \ldots, n$, $\ell = 1, 2, \ldots, M$, are arbitrary complex parameters. The symbols $\dagger$ and T appearing in the superscript indicate the transpose conjugate and transpose of the matrix, respectively, while M and n represent the component number and soliton number, respectively. The proof for the above bright n-soliton solution (4) can be obtained by verifying that the bilinear equations (3) satisfy the Jacobi identity [11]. One can also ascertain the integrability of the system by the existence of n-soliton solution, with arbitrary n.

3.1 *Bright one-soliton solution*

Here, we write the explicit form of bright one-soliton solution of the M-LSRI system (1), resulting for the choice $n = 1$ in eq. (4), as follows:

$$S^{(\ell)} = A_\ell \sqrt{k_{1R}\omega_{1R}} \operatorname{sech}\left(\eta_{1R} + \frac{R}{2}\right) e^{i(\eta_{1I} + \frac{\pi}{2})}, \quad \ell = 1, 2, ..., M, \tag{5a}$$

$$L = -2k_{1R}^2 \operatorname{sech}^2\left(\eta_{1R} + \frac{R}{2}\right), \tag{5b}$$

where

$$A_\ell = \alpha_1^{(\ell)} \left(\sum_{\ell=1}^{M} |\alpha_1^{(\ell)}|^2 \right)^{-1/2},$$

$$\mathrm{e}^R = \frac{-\sum_{\ell=1}^{M} |\alpha_1^{(\ell)}|^2}{4k_{1\mathrm{R}}\omega_{1\mathrm{R}}}, \quad \eta_{1R} = k_{1\mathrm{R}}x + (2k_{1\mathrm{R}}k_{1\mathrm{I}} - \omega_{1\mathrm{R}})y + \omega_{1\mathrm{R}}t$$

and

$$\eta_{1\mathrm{I}} = k_{1\mathrm{I}}x - (k_{1\mathrm{R}}^2 - k_{1\mathrm{I}}^2 + \omega_{1\mathrm{I}})y + \omega_{1\mathrm{I}}t.$$

In eq. (5), the subscript R (I) appearing in a particular complex parameter denotes the real (imaginary) part of that complex parameter. The above bright one-soliton solution is characterized by $(M+2)$ arbitrary complex parameters ($\alpha_1^{(\ell)}$, $\ell = 1, 2, ..., M$, k_1 and ω_1) and it becomes singular (non-singular) for $\mathrm{e}^{\mathrm{R}} < 0$ ($\mathrm{e}^{\mathrm{R}} > 0$). So, one can obtain the regular solitons when the condition $e^{\mathrm{R}} > 0$ is satisfied, which restricts one of the parameters among $k_{1\mathrm{R}}$ and $\omega_{1\mathrm{R}}$ to be negative while the other takes positive values.

The amplitude (peak value) of soliton in the LW component (L) is $2k_{1\mathrm{R}}^2$ and that of the ℓth SW component ($S^{(\ell)}$, $\ell = 1, 2, ..., M$) is $A_\ell\sqrt{k_{1\mathrm{R}}\omega_{1\mathrm{R}}}$. As the amplitude of the soliton in the LW component is independent of $\alpha_1^{(\ell)}$ and ω_1 parameters, one can control the soliton in the SW component by tuning these parameters without affecting the soliton in the LW component. Soliton of the present (2+1)D M-LSRI system can propagate in two planes, namely (x–y) and (x–t) planes with different velocities $((\omega_{1\mathrm{R}}/k_{1\mathrm{R}}) - 2k_{1\mathrm{I}})$ and $-(\omega_{1\mathrm{R}}/k_{1\mathrm{R}})$, respectively, for fixed t and y. By tuning the $k_{1\mathrm{I}}$ parameter one can alter the velocity of propagating bright soliton in the (x–y) plane without affecting the

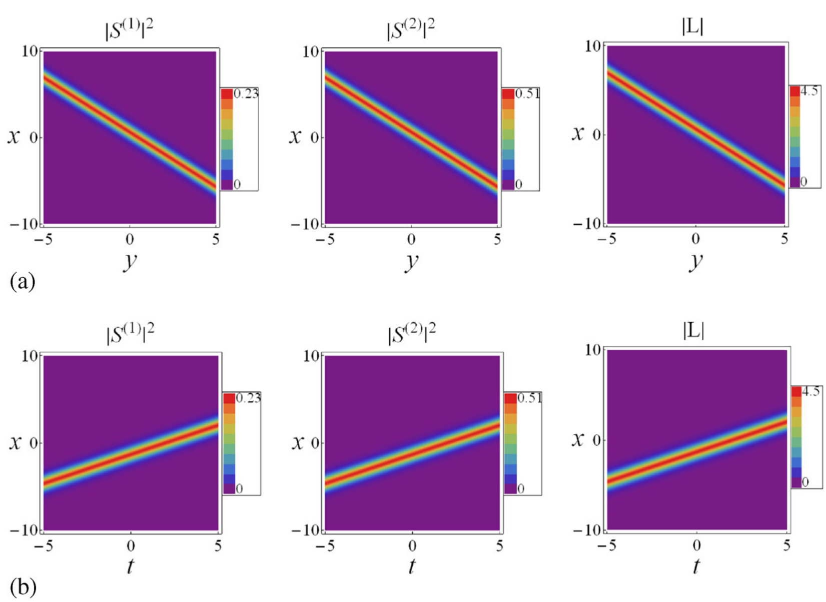

Figure 1. Propagation of bright one-soliton of the 2-LSRI system in the (x–y) plane for $t = 1$ (**a**) and in the (x–t) plane for $y = 1$ (**b**).

soliton velocity in the (x–t) plane. We have shown the propagation of bright one-soliton of 2-LSRI system in figure 1 for $k_1 = 1.5 + 0.3i, \omega_1 = -1 - 2i, \alpha_1^{(1)} = 1$ and $\alpha_1^{(2)} = 1.5$.

3.2 *Bright two-soliton solutions and their collisions*

Bright multisolitons of the present system show interesting collision properties with energy sharing (energy-exchange or shape-changing) phenomenon, similar to the vector solitons in multicomponent Manakov system, coupled Gross–Pitaevskii equations, etc. [17–23]. In order to understand this clearly, we consider the simple case of n-soliton solution, i.e., two-soliton solution ($n = 2$ in eq. (4)) of eq. (1) and analyse its dynamics. As the solitons in the present (2+1)D M-LSRI system admit different velocities in the (x–y) and (x–t) planes, they show different collision characteristics in those planes. Particularly, the solitons can undergo both head-on and overtaking collisions in the (x–y) plane for different soliton parameters. As the condition for non-singular solution restricts the velocity of solitons ((ω_{1R}/k_{1R}) and(ω_{2R}/k_{2R})) to be either positive or negative simultaneously, the solitons can undergo only overtaking collisions in the (x–t) plane. The bright solitons appearing in both components of the 1-LSRI system (1SW and 1LW) exhibit only elastic collision. However, they undergo energy sharing collisions if there are two or more SW components, i.e., M-LSRI system with $M \geq 2$.

From a detailed asymptotic analysis [11], change in the amplitude of a given jth soliton after collision in the ℓth SW component ($A_j^{(\ell)+}$) can be related to the amplitude of that soliton before collision ($A_j^{(\ell)-}$) in terms of the transition amplitudes ($T_j^{(\ell)}$) as

$$A_j^{(\ell)+} = T_j^{(\ell)} A_j^{(\ell)-}, \quad j = 1, 2, \quad \ell = 1, 2, ..., M, \tag{6}$$

where

$$T_1^{(\ell)} = \frac{1 - \lambda_1}{\sqrt{1 - \lambda_1 \lambda_2}} \left(\frac{(k_1 - k_2)(k_2 + k_1^*)}{(k_1^* - k_2^*)(k_2^* + k_1)} \right)^{1/2}$$

and

$$T_2^{(\ell)} = \frac{\sqrt{1 - \lambda_1 \lambda_2}}{1 - \lambda_2} \left(\frac{(k_2 + k_1^*)(k_1^* - k_2^*)}{(k_2^* + k_1)(k_1 - k_2)} \right)^{1/2},$$

in which

$$\lambda_1 = \frac{\alpha_2^{(\ell)} \kappa_{12}}{\alpha_1^{(\ell)} \kappa_{22}} \quad \text{and} \quad \lambda_2 = \frac{\alpha_1^{(\ell)} \kappa_{21}}{\alpha_2^{(\ell)} \kappa_{11}},$$

where the form of κ_{ij}, $i, j = 1, 2$, is as given in eq. (4c) for $n = 2$. The solitons undergo elastic collision for a special choice of soliton parameters ($\alpha_j^{(\ell)}$, $j = 1, 2$, $\ell = 1, 2, ..., M$) satisfying the condition

$$\frac{\alpha_1^{(1)}}{\alpha_2^{(1)}} = \frac{\alpha_1^{(2)}}{\alpha_2^{(2)}} = \cdots = \frac{\alpha_1^{(M)}}{\alpha_2^{(M)}},$$

for which $T_j^{(\ell)}$ become unimodular, i.e., $|T_j^{(\ell)}|^2 = 1$. However, the solitons appearing in the LW component undergo only elastic collision for all the choices of $\alpha_j^{(\ell)}$. Additionally,

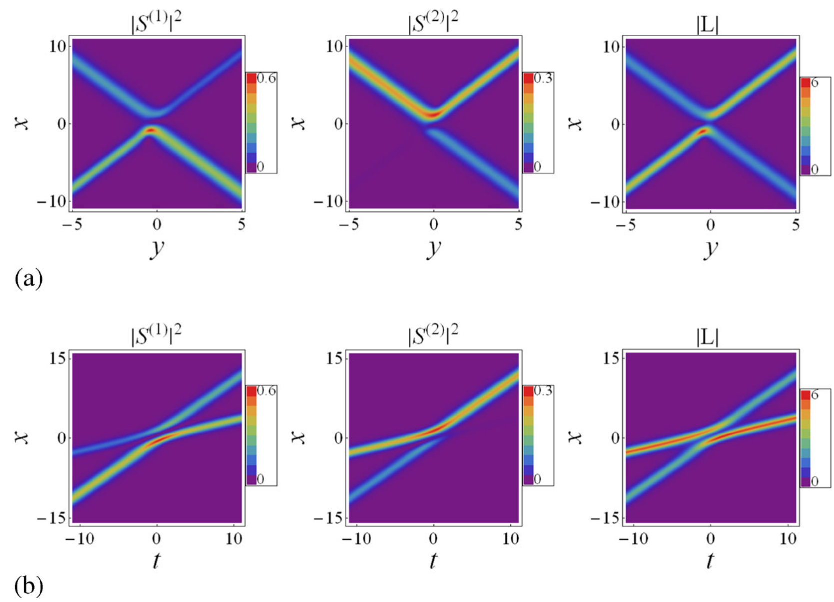

Figure 2. Energy sharing collision of two bright solitons in the 2-LSRI system. Head-on collision of solitons in the (x–y) plane at $t = 1$ (**a**) and overtaking collision of solitons in the (x–t) plane at $y = 1$ (**b**).

the soliton (say $s_j,\ j = 1, 2$) appearing in all the components experiences a phase-shift (Φ_j) given by

$$\Phi_1 = \ln\left(\sqrt{1-\lambda_1\lambda_2}\left|\frac{k_1-k_2}{k_1+k_2^*}\right|\right) \equiv -\Phi_2.$$

The energy sharing collision scenario of two bright solitons is shown in figure 2 for $k_1 = 1 + 0.3i$, $k_2 = 1.5 - i$, $\omega_1 = -1 - i$, $\omega_2 = -0.5 - 0.5i$, $\alpha_1^{(1)} = 2$, $\alpha_2^{(1)} = 1$, $\alpha_1^{(2)} = 1$, $\alpha_2^{(2)} = 0.08$. In the (x–y) plane, the amplitude of soliton s_1 (s_2) is enhanced (suppressed) in $S^{(1)}$ while the amplitude of soliton s_1 (s_2) gets suppressed (enhanced) in $S^{(2)}$. The switching nature of soliton intensity (energy) in the (x–t) plane is opposite to the switching phenomenon in the (x–y) plane. However, in both (x–y) and (x–t) planes, the LW solitons emerge unaltered after collision except for a phase-shift.

4. Dark solitons

As noted in §2, dark soliton solutions of the M-LSRI system (1) result for the choice $\lambda \neq 0$ in the bilinear eqs (3). In the following, we obtain the dark one- and two-soliton solutions of system (1) by applying the Hirota's bilinearization method [24–26].

4.1 *Dark one-soliton solution*

To construct the dark one-soliton solution, we choose the form of $g^{(\ell)}$ and f as $g^{(\ell)} = g_0^{(\ell)}(1 + \chi^2 g_2^{(\ell)})$, $\ell = 1, 2, \ldots, M$, and $f = 1 + \chi^2 f_2$. By substituting these expressions in the bilinear equations (3) and recursively solving the resulting set of equations, we get the explicit expressions for $g^{(\ell)}$ and f as $g^{(\ell)} = \tau_\ell(1+\mu_1^{(\ell)}\mathrm{e}^{\eta_1})\mathrm{e}^{i\psi_\ell}$, $\ell = 1, 2, ..., M$, and $f = 1 + \mathrm{e}^{\eta_1}$. Hence from eq. (2), the dark one-soliton solution can be written as

$$S^{(\ell)} = \frac{\tau_\ell}{2}\left[(1+\mu_1^{(\ell)}) - (1-\mu_1^{(\ell)})\tanh(\eta_1/2)\right]\mathrm{e}^{i\psi_\ell}, \quad \ell = 1, 2, ..., M, \tag{7a}$$

$$L = -\frac{k_1^2}{2}\,\mathrm{sech}^2(\eta_1/2), \tag{7b}$$

where

$$\eta_1 = k_1x + p_1y + \omega_1 t, \quad \psi_\ell = a_\ell x + b_\ell y + c_\ell t,$$

$$\lambda = \sum_{\ell=1}^{M} |\tau_\ell|^2$$

and

$$\mu_1^{(\ell)} = \frac{2a_\ell k_1 - p_1 - \omega_1 + ik_1^2}{2a_\ell k_1 - p_1 - \omega_1 - ik_1^2}.$$

Here a_ℓ, b_ℓ, c_ℓ, k_1, p_1 and ω_1 are real parameters, while τ_ℓ are complex parameters and they should satisfy the relation

$$\frac{4k_1^3}{\omega_1}\sum_{\ell=1}^{M}\frac{|\tau_\ell|^2}{(2a_\ell k_1 - p_1 - \omega_1)^2 + k_1^4} = 1$$

and $c_\ell = a_\ell^2 - b_\ell$, $\ell = 1, 2, ..., M$. The dark one-soliton solution (7) is characterized by $(4M+2)$ arbitrary real parameters.

The absolute square of the SW solutions and the absolute of the LW solution given by the above equation (7) can be written in a compact form as

$$|S^{(\ell)}|^2 = |\tau_\ell|^2\left[1 - A_\ell\,\mathrm{sech}^2(\eta_1/2)\right], \quad \ell = 1, 2, ..., M, \tag{8a}$$

$$|L| = \frac{k_1^2}{2}\,\mathrm{sech}^2, \tag{8b}$$

where

$$A_\ell = \frac{k_1^4}{(2a_\ell k_1 - p_1 - \omega_1)^2 + k_1^4}$$

determines the degree of darkness of dark soliton in the ℓth SW component and $|\tau_\ell|^2$ represents its background intensity. Depending upon the values of A_ℓ, one can get the dark and gray solitons in the SW components, i.e., $A_\ell = 1$ and $A_\ell < 1$ result in dark and gray solitons, respectively. On the other hand, the LW component always results in bright soliton with amplitude $k_1^2/2$, irrespective of other soliton parameters. The velocity of soliton (bright in LW and dark in SW) is $-p_1/k_1$ in the (x–y) plane and $-\omega_1/k_1$ in the (x–t) plane. So the directions of soliton propagation (velocities) in both planes can be made different by controlling these quantities. We have shown a typical dark (bright) soliton propagation appearing in the SW (LW) component of the 2-LSRI system in figure 3 for $k_1 = 4$, $p_1 = -2$, $a_1 = 1$, $a_2 = 1.2$, $b_1 = 1.1$, $b_2 = 1.3$, $\tau_1 = 2$ and $\tau_2 = 1$.

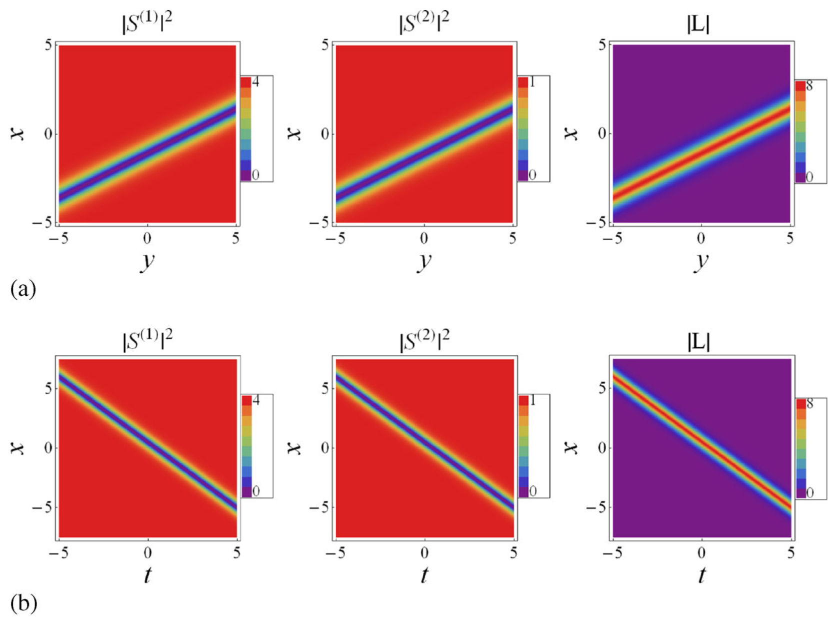

Figure 3. Propagation of dark one-soliton in 2-LSRI system in (*x*–*y*) plane for $t = 1$ (**a**) and in (*x*–*t*) plane for $y = 1$ (**b**).

4.2 *Dark two-soliton solutions and their collisions*

The dark two-soliton solution can be constructed by restricting the power series expansions for $g^{(\ell)}$ and f as $g^{(\ell)} = g_0^{(\ell)}(1 + \chi^2 g_2^{(\ell)} + \chi^4 g_4^{(\ell)})$, $\ell = 1, 2, ..., M$, and $f = 1 + \chi^2 f_2 + \chi^4 f_4$. The explicit forms of $g^{(\ell)}$ and f can be obtained as

$$g^{(\ell)} = \tau_\ell \left(1 + \mu_1^{(\ell)} e^{\eta_1} + \mu_2^{(\ell)} e^{\eta_2} + \mu_1^{(\ell)} \mu_2^{(\ell)} \Omega e^{\eta_1+\eta_2}\right) e^{i\psi_\ell}, \quad \ell = 1, 2, ..., M, \tag{9a}$$

$$f = 1 + e^{\eta_1} + e^{\eta_2} + \Omega e^{\eta_1+\eta_2}, \tag{9b}$$

where

$$\eta_j = k_j x + p_j y + \omega_j t, \quad \psi_\ell = a_\ell x + b_\ell y + c_\ell t,$$

$$\lambda = \sum_{\ell=1}^{M} |\tau_\ell|^2, \quad \mu_j^{(\ell)} = \frac{2a_\ell k_j - p_j - \omega_j + ik_j^2}{2a_\ell k_j - p_j - \omega_j - ik_j^2}, \; j = 1, 2, \quad \ell = 1, 2, ..., M$$

and

$$\Omega = \frac{k_1^2 k_2^2 (k_1 - k_2)^2 + (k_1(p_2 + \omega_2) - k_2(p_1 + \omega_1))^2}{k_1^2 k_2^2 (k_1 + k_2)^2 + (k_1(p_2 + \omega_2) - k_2(p_1 + \omega_1))^2}.$$

The above two-soliton solution is characterized by $(3M + 6)$ real parameters a_ℓ, b_ℓ, c_ℓ, k_j, p_j and ω_j, and M complex parameters τ_ℓ, with $(M + 2)$ relations

$$c_\ell = a_\ell^2 - b_\ell, \quad \ell = 1, 2, ..., M$$

and

$$\frac{2}{\omega_j k_j}\sum_{\ell=1}^{M}|\tau_\ell|^2\left(1-\text{Re}[\mu_j^{(\ell)}]\right)=1, \quad j=1,2.$$

Hence there are only $(4M+4)$ number of arbitrary real constants. The velocities and darkness (amplitude) of dark (bright) solitons appearing in the SW (LW) components can be controlled by tuning these arbitrary parameters.

The collision dynamics of dark solitons can be explored by performing an asymptotic analysis, as was done for the bright soliton collision process, which we have not discussed in this paper here on considering the length of the article. From the asymptotic analysis, we find that the dark solitons appearing in the SW components undergo only elastic collision for all choices of soliton parameters, in contrast to the energy sharing collision of bright solitons in the SW components. Also, the bright solitons appearing in the LW components exhibit the usual elastic collision. But these colliding solitons experience phase-shift. By tuning the soliton parameters, one can demonstrate that the collision among two dark/gray solitons or collision between a dark and a gray solitons in the SW components is elastic. Thus, irrespective of the nature of dark-soliton profile (either dark or gray) their amplitudes (intensities) remain unaltered after collision. Such an elastic collision of solitons (a dark and gray solitons in SW component and two bright solitons in the LW component) of the 2-LSRI system is given in figure 4 for $k_1 = 1.5$, $k_2 = 2.5$, $p_1 = -1.6$, $p_2 = 2.6$,

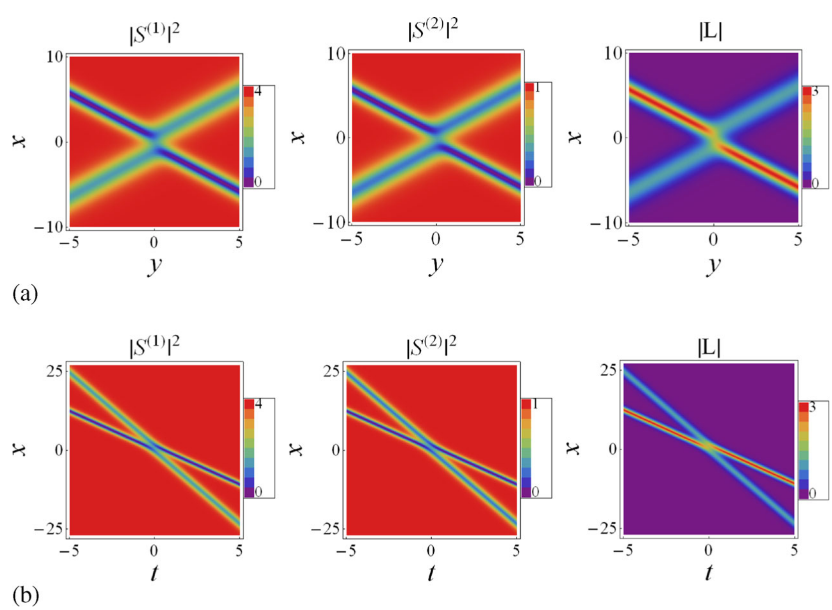

Figure 4. Elastic collision of bright (dark–gray) solitons in the LW (SW) component(s) of 2-LSRI system in the $(x$–$y)$ plane at $t = 0.25$ (**a**) and in the $(x$–$t)$ plane at $y = -0.25$ (**b**).

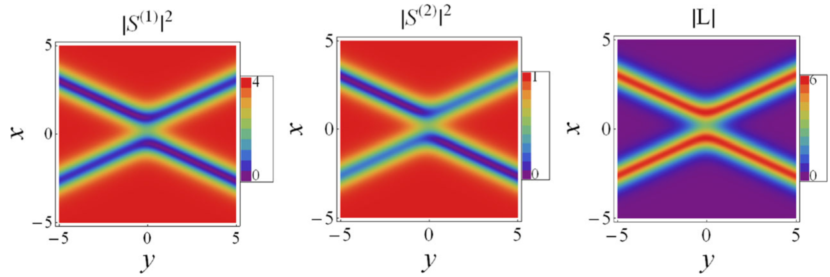

Figure 5. Elastic collision of dark–dark, dark–gray and bright–bright solitons in the $S^{(1)}$, $S^{(2)}$ and L components of the 2-LSRI system in the $(x\text{–}y)$ plane for $k_1 = 3.5$, $k_2 = 3.5$, $p_1 = -1.6$ and $p_2 = 1.6$, with other parameters same as in figure 4.

$a_1 = 1$, $a_2 = 1.2$, $b_1 = 1.1$, $b_2 = 1.3$, $\tau_1 = 2$ and $\tau_2 = 1$. Also, in figure 5 we have shown the elastic collision of dark–dark ($S^{(1)}$-component), dark–gray ($S^{(2)}$-component) and bright–bright (L-component) solitons of the 2-LSRI system.

Through dark–dark soliton collision process, one can form a bound state for the same velocity dark solitons with coinciding (different) central position(s) resulting in single (double) well type structures which propagate like a single soliton (parallel solitons). Also, we wish to emphasize that these dark soliton bound states do not admit periodic oscillations as in the case of bright/bright–dark solitons [12,13]. This procedure can be generalized to construct the dark multisoliton solution in a straightforward manner which involve very lengthy and tedious calculations, and the details will be presented elsewhere.

5. Conclusions

We have considered an integrable multicomponent long wave–short wave resonance interaction (M-LSRI) equation governing the dynamics of nonlinear interaction between multiple (M) short waves and a long wave in the context of nonlinear optics. To unravel the interesting propagation dynamics of multicomponent plane solitons we have constructed soliton solutions by using the Hirota's bilinearization method. We have briefly revisited the earlier results on the bright multisoliton solution and demonstrated the fascinating propagation dynamics and collision processes. Particularly, we have shown that the amplitude of bright soliton appearing in the short-wave components can be controlled by tuning the polarization parameters without affecting the amplitude of soliton appearing in the long-wave component. From the collision dynamics of solitons in the M-LSRI system, we have identified the interesting energy sharing collision of bright solitons in the short-wave components when $M \geq 2$. The solitons in the short-wave component (for special choices of polarization parameters) can also undergo elastic collision accompanied by a phase-shift. From the dark one-soliton solution, we have observed that the nature of soliton profile (dark or gray) in the short-wave component can be controlled by tuning the soliton parameters, whereas the long-wave component supports only bright solitons. Analysis on the dark two-soliton solution reveals that the dark solitons always exhibit only elastic collisions with a phase-shift. Also, a collision between two dark/gray solitons or a collision between dark and gray solitons is also shown to be elastic. As a future study, one can construct the dark multisoliton solution by generalizing the present algorithm and investigate the underlying dynamics.

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